

Chapter: Trigonometry Basics

Introduction

Trigonometry is a branch of mathematics that studies the relationships between the angles and sides of triangles, particularly right-angled triangles. It is an essential tool in many fields such as physics, engineering, architecture, and navigation. The word “trigonometry” comes from the Greek words *trigonon* (triangle) and *metron* (measure).

In this chapter, we will explore the fundamental concepts of trigonometry, focusing primarily on the right-angled triangle. We will learn how to define trigonometric ratios, understand their properties, and apply them to solve problems involving angles and sides.

1. Right-Angled Triangle and Terminology

A **right-angled triangle** is a triangle where one of the angles is exactly 90° . The sides of a right-angled triangle have specific names based on their position relative to the angle of interest (usually denoted as θ):

- **Hypotenuse:** The side opposite the right angle and the longest side of the triangle.
 - **Opposite side:** The side opposite to the angle θ .
 - **Adjacent side:** The side next to the angle θ , excluding the hypotenuse.
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2. Trigonometric Ratios

The three primary trigonometric ratios are defined as the ratios of the lengths of the sides of a right-angled triangle:

$$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}} \quad \cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}} \quad \tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

Important Note:

- These ratios are only valid for angles between 0° and 90° in a right-angled triangle.
 - Each ratio corresponds to a function that relates an angle to the ratio of two sides.
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3. Reciprocal Ratios

Besides the three primary ratios, there are three reciprocal trigonometric ratios:

$$\csc\theta = \frac{1}{\sin\theta} = \frac{\text{Hypotenuse}}{\text{Opposite side}} \quad \sec\theta = \frac{1}{\cos\theta} = \frac{\text{Hypotenuse}}{\text{Adjacent side}} \quad \cot\theta = \frac{1}{\tan\theta} = \frac{\text{Adjacent side}}{\text{Opposite side}}$$

4. Fundamental Identities

The Pythagorean theorem applies to right-angled triangles:

$$(\text{Hypotenuse})^2 = (\text{Opposite side})^2 + (\text{Adjacent side})^2$$

Using trigonometric ratios, this leads to the fundamental identity:

$$\sin^2\theta + \cos^2\theta = 1$$

From this identity, we also get:

$$1 + \tan^2\theta = \sec^2\theta \quad 1 + \cot^2\theta = \csc^2\theta$$

5. Using Trigonometric Ratios to Find Sides and Angles

Finding a side

Given one angle θ (other than 90°) and one side, you can find the other sides using the trigonometric ratios:

- To find the opposite side:

$$\text{Opposite} = \text{Hypotenuse} \times \sin\theta$$

or

$$\text{Opposite} = \text{Adjacent} \times \tan\theta$$

- To find the adjacent side:

$$\text{Adjacent} = \text{Hypotenuse} \times \cos\theta$$

or

$$\text{Adjacent} = \frac{\text{Opposite}}{\tan\theta}$$

Finding an angle

If two sides are known, the angle θ can be found by using the inverse trigonometric functions:

$$\theta = \sin^{-1}\left(\frac{\text{Opposite}}{\text{Hypotenuse}}\right) \quad \theta = \cos^{-1}\left(\frac{\text{Adjacent}}{\text{Hypotenuse}}\right) \quad \theta = \tan^{-1}\left(\frac{\text{Opposite}}{\text{Adjacent}}\right)$$

6. Solving Right-Angled Triangle Problems

Step 1: Identify the known sides or angles.

Step 2: Choose the appropriate trigonometric ratio.

Step 3: Write the equation based on the ratio.

Step 4: Solve for the unknown value.

Step 5: Verify the answer with the triangle properties.

7. Example Problems

Example 1:

A right triangle has an angle $\theta = 30^\circ$ and hypotenuse 10 cm. Find the length of the side opposite θ .

Solution:

$$\text{Opposite} = \text{Hypotenuse} \times \sin 30^\circ = 10 \times \frac{1}{2} = 5 \text{ cm}$$

Example 2:

In a right-angled triangle, the side adjacent to $\theta = 45^\circ$ is 7 cm. Find the hypotenuse.

Solution:

$$\text{Hypotenuse} = \frac{\text{Adjacent}}{\cos 45^\circ} = \frac{7}{\frac{\sqrt{2}}{2}} = 7 \times \frac{2}{\sqrt{2}} = 7\sqrt{2} \approx 9.9 \text{ cm}$$

Summary

- Trigonometry studies the relationship between the sides and angles of triangles.
 - The three basic trigonometric ratios are sine, cosine, and tangent.
 - Reciprocal functions include cosecant, secant, and cotangent.
 - The Pythagorean theorem forms the foundation for fundamental identities.
 - Trigonometric ratios are used to calculate unknown sides or angles in right-angled triangles.
 - Inverse trigonometric functions help find angles when side lengths are known.
 - Practical problem-solving involves identifying known data, choosing the correct ratio, and solving systematically.
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